Black Scholes and Binomial Option Pricing Problems

1. Employee Stock Options

Gary Levin is the CEO of Moutainbrook Trading Company. The board of directors has just granted Mr. Levin 20,000 at-the-money European call options on the company's stock., which is currently trading at $50 per share. The stock pays no dividends. The options will expire in 4 years, and the standard deviation of the returns on the stock is 55%. Treasury bills that mature in 4 years currently yield a continuously compounded interest rate of 6^.

a. Use the Black-Scholes model to calculate the value of the stock options.

$d_1 = [\ln(S/E) + [R + \sigma^2/2)t]/\sqrt{\sigma^2 t}$

$d_1 = \ln(\$50/\$50) + [.06 + (.55)^2/2 \text{ x } (4)]/(.55 \text{ x } \sqrt{2} = .7682$

$d_2 = d_1 - \sqrt{\sigma^2 t}$

$d_2 = .7682 - (.55 \text{ x } \sqrt{4} = -.3318$

$N(d_1) = .7788$

$N(d_2) = .3700$

$C = \$50(.7788) – (\$50\ e^{-(.06)(4)})(.3700) = \$24.39$

Since the option grant is for 20,000 options, the value of the grant is:

Grant Value = 20,000($24.39) = $487,747.66

b. You are Mr. Levin's financial advisor. He must choose between the previously mentioned stock option package and an immediate $450,000 bonus. If he is risk-neutral, which would you recommend?

Since he is risk-neutral, you should recommend the alternative that has the highest net present value. Since the expected value of the stock option package is worth more than $450,000, he would prefer to be compensated with the options rather than the immediate bonus.

c. How would your answer to (b) change if Mr. Levin were risk-averse and he Could not sell the options prior to expiration?

If he is risk-averse, he may or may not prefer the stock option package to the immediate bonus. Even though the stock option package has a higher net present value, he may not prefer is because it is undiversified. The fact that he cannot sell his options prematurely makes it much more risky than the immediate bonus.

3. Binomial Model

Gasworks, Inc. has been approached to sell up to 5 million gallons of gasoline in 3 months at a price of $1.85 per gallon. Gasoline is currently selling on the wholesale market at $1.65 per gallon and has a standard deviation of 46%. If the risk-free rate is 6% per year, what is the value of this option?

Since the contract is to sell up to 5 million gallons, it is a call option, so we need to value the contract accordingly. Using the binomial mode, we will find the value of u and d, which are:

$u = e^{\sigma/\sqrt{n}} = e^{.46/\sqrt{4}} = 1.26$ [since n = number of intervals in a year,i.e.12/3]

$d = 1/u = 1/1.26 = .79$

This implies the % increase if gasoline increases will be 26%, and the % decrease if prices fall will be 21% [1-.79]. So, the price in 3 months with an up or down move will be:

$P_{up} = \$1.65(1.26) = \$2.08$

$P_{Down} = \$1.65(.79) = \$1.31$

The option is worthless if the price decreases. On the other hand, if the price increases, the value of the option per gallon is:

Value with price increase: $2.08 - $1.85 = $.23

Next, we need to find the risk neutral probability of a price increase or decrease, which will be:

Quarterly Return = .06/4 = .015 = Expected Value of the ΔGas Prices

.015 = Prob[rise in prices] {$.26} + [1-Prob[rise in prices]]{-.21}
.015 + .21 = [.26 + .21] {Prob[rise in prices]}
.225/.47 = .4787 = Prob.[rise in prices]
1-.4787 = .52127 = Prob{decrease in prices]

The contract will not be exercised if gasoline prices fall, so the value of the contract with a price decrease is 0.

C = [.4787($.23) + (.52127)($0)]/ {1 + .06/4} = $.01101/1.015 = $.10847

Consequently, the value of the entire contract is: $.10847 x 5 million =$542,369

6. Real Options
Sardano and Sons is a large, publicly held company that is considering leasing a warehouse. One of the company's divisions specializes in manufacturing steel, and this particular warehouse is the only facility in the area that suits the firm's operations. The current price of steel is $3,600 per ton. If the price of steel falls over the next six months, the company will purchase 400 tons of steel and produce 4,800 steel rods. Each steel rod will cost $120 to manufacture, and the company plans to sell the rods for $360 each. It will take only a matter of days to produce and sell the steel rods. If the price of steel rises or remains the same, it will not be profitable to undertake the project, and the company will allow the lease to expire without producing any steel rods. Treasury bills that mature in six months yield a continuously compounded interest rate of 4.5%, and the standard deviation of the returns on steel is 45%. Use the Black-Scholes model to determine the maximum amount that the company should be willing to pay for the lease.

When solving a question dealing with real options, begin by identifying the option-like features of the situation. First, since Sardano will only choose to manufacture the steel rods if the price of steel falls, the lease, which gives the firm the ability to manufacture steel, can be viewed as a put option. Second, since the firm will receive a fixed amount of money if it chooses to manufacture the rods:

Amount received = 4,800 rods x ($360 - $120) = $1,152,000

The amount received can be viewed as the put option's strike price (K). Third, since the project requires Sardano to purchase 400 tons of steel and the current price of steel is $3,600/ton, the current price of the underlying asset (S) to be used in the Black-Scholes formula is:

"Stock" price = 400 tons x $3,600/ton = $1,440,000

Finally, since Sardano must decide whether to purchase the steel or not in 6 months, the firm's real option to manufacture steel rods can be viewed as having a time to expiration (t) of 6 months or ½ a year. In order to calculate the value of this real put option, we can use the Black-Scholes model to determine the value of an otherwise identical call option and then infer the value of the put using the put-call parity relationship.

Using the Black-Scholes model to determine the value of the call option we find:

$d_1 = [\ln(S/K) + [R + \sigma^2/2)t]/\sqrt{\sigma^2 t}$

$d_1 = \ln(\$1,440,000/\$1,152,000) + [.045 + (.45)^2/2 \times (1/2)]/(.45 \times \sqrt{1/2} = .9311$

$d_2 = d_1 - \sqrt{\sigma^2 t}$

$d_2 = .9311 - (.45 \times \sqrt{1/2} = .6129$

$N(d_1) = .8241$

$N(d_2) = .7300$

$C = \$1,440,000(..8241) – (\$1,152,000\ e^{-(.045)(1/2)})(.7300) = \$364,419.87$

The value of the call. Therefore the value of the put option may be found from:

$C = P + S - Ke^{-Rt}$
$\$364,419.87 = P + \$1,440,000 - \$1,152,000e^{-.045(1/2)}$
$P = \$50,789.29$
The most the company should be willing to pay for the lease option.